
March 2009
Regulars

Outer space: When errors snowball

by John D. Barrow

Errors cause problems. But errors are not necessarily mistakes. They are uncertainties in our knowledge of the true state of affairs. Sometimes these are one−offs: we want to measure a baby's weight and our digital scales are only accurate to a gram, so we have an unavoidable "error" of one gram in our determination of the baby's weight. Sometimes errors can be dramatically cumulative, and double (or worse) at each stage in a many−step process: this is the source of the phenomenon of *chaos* that has been so well publicised over the past thirty years. It bedevils our attempts to predict the weather with great accuracy. In between these two extremes there is another type of error that stays the same at each stage in a many−step process, but accumulates in ways that can lead to a significant overall uncertainty.

Athletics track: accuracy matters

If you are building a swimming pool or a running track that is going to be used for races and time trials covering many laps, then the accuracy of the lap length is very important. Make the track or pool too short and the distance covered will fall short of the required distance more and more as the race progresses. Any records set will be invalid when ultimately checked by laser ranging. A running track can make adjustments to its finish line to counter final construction errors, but a swimming pool doesn't have that option.

Suppose a race has length R and covers N laps of length L, so $NL = R$. If the construction of the stadium has led to an error in the length of a lap equal to ϵ, then the cumulative error over the race will be a distance $N\epsilon = \epsilon R/L$. In practice, it is timing that we are interested in for record purposes. If the time to complete the race is T, then the average speed is R/T and the overall time error, ΔT, introduced by the error in the length of the lap is $\Delta T = (R/L \times \epsilon)/(R/T) = T\epsilon/L$, so we have the very simple result:

$$\Delta T/T = \epsilon/L.$$

This tells us that the fractional error in the total race time is equal to the fractional error in the lap length.

In the construction of international athletics tracks the IAAF tolerance standard is that a 400 metre lap determined by an average of measures of its straights and bends is allowed to be up to 4 cm over length but not under length. For FINA approved swimming pools, a 50 metre pool can be built up to 3 cm over length, but again there is no tolerance for it to be under length. This is to ensure records can't be broken by under length races (or, worse still, that all times recorded on a short track or in a short pool are invalid for any statistical or comparative ranking purposes).

	Lap length ***L***	**Max allowed lap length error**	**Overall race time error in seconds if race lasts *T* seconds**	**Duration of race with > 0.01 sec error**
Athletics track	400m	0.04m	10^{-4} ×(T/1sec)	100 sec
Long−course swimming	50m	0.03m	6 × 10^{-4}× (T/1sec)	16.7 sec
Short−course swimming	25m	0.03m	12 × 10^{-4} ×(T/1sec)	8.3 sec

In the table above we have worked out the consequences of this formula if the maximum tolerance is used, and the running track is 4 cm over length and the swimming pool is 3 cm over length. In the last column we work out the amount of time the race would have to go on for in order for the lap length error to produce an overall timing error exceeding 0.01 sec, which is the accuracy of time keeping for records in athletics and swimming.

Precision is crucial. Computer model of the London 2012 Aquatics Centre. Image courtesy London 2012.

All swimming races exceeding 50m are affected and all standard track events of 800m and longer. For a 10,000 metre race at world record pace for men (26 mins 30 sec), the cumulative error would be 0.16 secs, quite significant. Of course, the situation in running events is less clear−cut because competitors can choose to run difference total distances by running wide of the inside lane kerb for tactical reasons, and almost everyone runs over distance. In swimming the situation is more clear−cut. Careful surveying really counts.

Plus is part of the family of activities in the Millennium Mathematics Project, which also includes the NRICH and MOTIVATE sites.