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Features



## Number crunching ants

by Liz Newton



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Georges Louis Leclerc Comte de Buffon

## Number crunching ants

What does a bread-throwing Frenchman have in common with a tiny species of ant with a brain smaller than a single grain of salt? You could be mistaken for thinking that this is the beginning of a bad joke, but it's a serious question and the intriguing answer has taken scientists and mathematicians years to find.

The story begins back in the eighteenth century, with a French mathematician called Georges Louis Leclerc, Comte de Buffon.

### Breadsticks, needles and pi

Buffon had a long-standing interest in the calculus of probability, and wanted to show that some probability calculations relied not on algebra, but on geometry. In 1777, he came up with an imaginative idea which showed just this.

Buffon invented a game in which two men throw stick-like loaves of French bread over their shoulders onto a wooden floor made up of parallel planks, betting on whether the loaves would fall over a crack between the planks.

While he was calculating the lengths the loaves would need to be to make the game fair, he discovered an extraordinary relationship: by finding the probability of a successful throw, he could statistically estimate the value of pi the constant representing the ratio of any given circle's circumference to its diameter.

Buffon went on to explain that he could do exactly the same thing by simply using a sheet of lined paper and a needle. By drawing equally-spaced parallel lines on a sheet of paper and randomly casting a needle onto it many times, Buffon showed that he could estimate pi using the probability of the needle touching a line. As long as the length of the needle was equal to or less than the distance between the lines, pi would be approximately equal to twice the total number of drops of the needle, divided by the number of successful "hits":

$$2 \frac{\text{total drops}}{\text{number of hits}} \approx \pi.$$

The statistical estimation of pi by this method became known as *Buffon's needle algorithm*.

The next twist in the story came much later, when scientists from the University of Bath were studying a certain species of ant which had thrown up a mystery all of its own.

### House-hunting ants

Imagine being alone in an enormous cave. You have to work out the area of the floor of the cave, but it's rocky and uneven, you have no tape measure and to top it all off, it's pitch black.

Even the most experienced engineer would struggle with such a task, but for ants of the species *Temnothorax albipennis*, it's a mathematical feat that they achieve every time they move house. In fact, these ants can measure and compare the areas of two or three potential nest sites or "caves" and pick the best one.

## Number crunching ants



Small? Maybe. Powerful? Definitely!

But how can such a tiny animal process the complex equations needed to measure and compare such vast areas, with no light to guide them? This is the question that puzzled the scientists studying the ants, and the extraordinary answer lay in the algorithm Buffon had proposed over 200 years before.

Ants of the species *Temnothorax albipennis* are only 2.5mm long, and live in small rock crevices in the coastal cliffs of the South of England. Colonies build nests inside these crevices, and the area of the nest is directly related to the number of ants in the colony. If the nest is destroyed or the colony grows too big, the ants need to migrate to a new site. Certain ants within the colony are designated as scouts, and it is their job to find potential new nest sites for the whole colony.

A scout will visit any potential nest site at least twice. On its first visit to a new site, a scout will spend around two minutes exploring the site. During these two minutes, the scout lays down a chemical scent trail known as a pheromone trail. This trail is specific to the individual ant, and each scout can recognise its own scent. The scout will then leave the nest site and return to its colony. The scientists think that this allows them to learn the route, so that they can direct their nest mates.

On the second visit to the site the scout will repeat its exploration of the area, but this time it does not lay down a trail. The scientists discovered that every time the scout encounters its first trail, it slows down, as if making a mental note of this encounter.

This comparison of the first and second exploration provides the key to the ants' estimation of the nest area. The scientists realised that in the same way that Buffon used geometric reasoning instead of algebra to calculate pi in his needle algorithm, the ants could use the geometry of their trails to estimate the area of a plane such as the nest floor.

This is because the estimated area of a plane ( $A$ ) is inversely proportional to the number of intersections ( $N$ ) between two sets of lines, of total lengths  $S$  and  $L$ , randomly scattered onto it. This can be expressed as the formula:

## Number crunching ants

$$A = \frac{2SL}{\pi N}.$$

In other words, nest area is inversely proportional to the intersection frequency the smaller the plane (or in this case the nest site), the more times the scout will encounter its first trail, and vice versa.

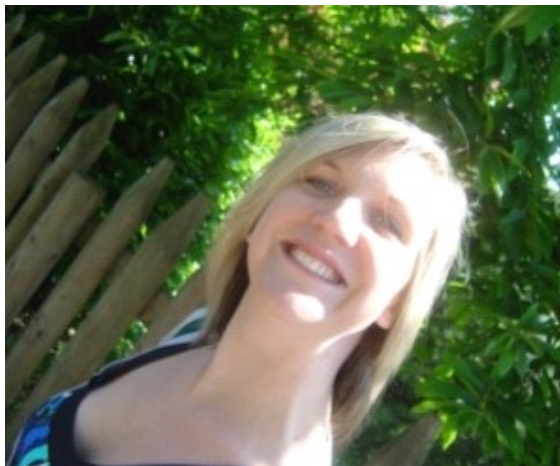
The scientists also found that the ants' trails, on both their first and second visits, were sufficiently random for this equation to work, and that the length of each scout's initial trail was approximately the same length in each potential nest site they visited, regardless of its size.

So the link between Buffon's bread-throwing and the ants' house-hunting was finally established. Does this mean that each tiny ant has the same mathematical capabilities as Buffon? Of course not and the scientists are quick to point this out. There is no evidence that the ants are aware of the value of pi, or that they calculate the distances involved in millimetres. The formula inspired by Buffon simply shows that the ants are capable of using a relatively simple rule of thumb to estimate whether a potential nest site is suitable for their colony's size.

Perhaps most importantly however, the link shows that mathematics can be found everywhere in nature. The numbers and symbols we use to understand maths are simply part of the language we have invented to understand the world around us. Looking at the extraordinary success of the ants, it's clear that to be good at maths, a big brain isn't quite as important as it might seem.

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## About this article



Liz has a biology degree and is currently studying for a Masters in Science Communication at UWE in Bristol. She also works for a wildlife television company, and in her spare time she enjoys watching *Hollyoaks* and playing with creepy crawlies.

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