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Features

The amazing librarian

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Given an $n \times n$ matrix H and a vector I of length n, their product J is also vector of length n (strictly speaking we should write I as a column vector here). The **first** entry of J is the sum:

the first entry of the **first** row of H Ã—â€‰ the first entry of I
+ the second entry of the **first** row of H Ã—â€‰ the second entry of I
+ ...
+ the last entry of the **first** row of H Ã—â€‰ the last entry of I.

Now recall how we have built the matrix H. The first row records the information on back links of the page P_1: the first entry of this row is zero if P_1 does not link to itself and $\frac{1}{l_1}$ if it does, the second entry of the first row is 0 if P_2 does not link to P_1 and $\frac{1}{l_2}$ if it does, etc. Here the l_j are the total number of links on page P_j.

Now each entry of the first row of H is multiplied by the corresponding entry of the importance vector I. So the first entry is multiplied by the importance of P_1, to get either 0 or $\frac{I_1}{l_1}$. The second entry is multiplied by the importance of P_2 to get either 0 or $\frac{I_2}{l_2}$, etc.

Putting all this together shows that the first entry of the vector J is equal to the sum of the importance of the pages that link to page P_1, weighted by the total number of links on those pages. This is precisely what we defined the importance of page P_1 to be. So the first entry of J is equal to the first entry of the importance vector I.

The **second** entry of J is the sum:

the first entry of the **second** row of H Ã—â€‰ the first entry of I
+ the second entry of the **second** row of H Ã—â€‰ the second entry of I
+ ...
+ the last entry of the **second** row of H Ã—â€‰ the last entry of I.

Again we see that the second entry of J is equal to I_2, the second entry of the importance vector I. The multiplication carries on correspondingly for each row of H, and this shows that $J = I$.

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